Sensor Network Design for Maximizing Reliability of Bilinear Processes

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The problem of selecting the variables to be measured in order to maximize process reliability was tackled in our previous articles (Ali and Narasimhan, 1993, 1995). In this article, this approach is extended to the optimal design of sensor networks for bilinear processes. Diverse processes, such as a mineral beneficiation plant, a separation system of a synthetic juice plant, and a crude preheat train of a refinery are used to illustrate the utility of this approach.

Introduction

The placement of sensors is a critical activity for synthesizing operating procedures, control schemes, and designing performance monitoring systems. It also forms an integral part of the activity of developing piping and instrumentation diagrams (P&IDs) for chemical plants. In spite of the fact that this problem poses a serious challenge to process design engineers, it has received scant attention in the literature. Vaclavek and Loucka (1976) first addressed the problem of sensor placement with the objective of ensuring the observability of all important process variables. Madron and Veverka (1992) and Bansal et al. (1994) have solved the problem of obtaining the least-cost sensor network while ensuring the observability of all variables. However, these objectives do not consider the reliability of sensor networks that may be affected by sensor failures. Ali and Narasimhan (1993, 1995) proposed the concept of reliability of a process variable that is the probability of estimating a variable for a given sensor network and specified sensor failure probabilities. It was shown that this concept provides a general quantitative measure for observability and redundancy. Based on this concept, a sensor network design algorithm called SENNET was developed for maximizing the least reliability among all variables. Independently, Turbatte et al. (1991) have also proposed a similar concept of system reliability that gives the probability that all variables are observable when sensors are likely to fail. These authors have also described a procedure for computing the system reliability for linear processes, although the design of an optimal sensor network has not been addressed. Ragot et al. (1992) have analyzed the problem of sensor placement to ensure the observability of all variables in a bilinear process, but reliability issues have not been considered.

In this article, we treat the problem of sensor network design for bilinear processes. A process is called bilinear if the steady-state equations that describe the process contain terms that are at most products of two variables. For example, in a plant a subsystem consisting of a train of distillation columns (or other separation systems) can be regarded as a bilinear process, if we consider only the flow and component balance equations. If the heat capacities of the streams are not dependent on the composition, then energy networks can also be treated as bilinear processes. Examples of such subsystems in a plant are steam distribution networks or heat exchanger networks (for example, a crude preheat train of a refinery). Thus, bilinear processes form an important class of processes.

In this article, we have designed a sensor network to maximize the network reliability that is defined as the least reliability among all mass and composition (or temperature) variables. We consider the case when only a minimum number of sensors is used, giving only one way of estimating every variable. The utility of the design algorithms we have developed is demonstrated through application to industrial processes such as a mineral beneficiation circuit, a subsystem consisting of a sequence of distillation columns, and a crude preheat train of a refinery.

Problem Definition and Assumptions

The problem we consider in this work is the optimal location of flow and composition sensors in multicomponent

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processes and the optimal location of flow and temperature sensors in energy-distribution or recovery networks. We assume that the failure probabilities of sensors of different streams have been specified, and we attempt to design sensor networks that maximize the network reliability.

The optimal sensor network design depends not only on the sensor failure probabilities, but also on the equations that relate the stream variables. For a multicomponent process, the process units that occur are mixers, splitters and separators. The equations that characterize these units are flow balances, component balances (or equality of compositions of streams incident on a splitter), and normalization equations. Since flow and composition variables occur as product terms in component balances, we refer to a multicomponent system as a bilinear process. In an energy-distribution subsystem, the process units are mixers, splitters, heat exchangers, heaters, and coolers, and the governing equations are flow balances and energy balances (or equality of the temperatures of the streams on a splitter). Although energy balances can be nonlinear due to the dependence of specific enthalpy on temperature, these processes can also be treated as bilinear systems for the purpose of sensor network design.

In order to solve the preceding problem in sufficient generality, it is necessary to identify all possible ways of estimating each variable for a given sensor network, so that the reliability of estimating every variable can be computed (Ali and Narasimhan, 1995). However, due to the different type of equations relating the variables as well as the bilinear terms, this becomes a complex problem. As a preliminary effort in this direction, we make the following assumptions in order to make the problem tractable:

1. We only consider the problem of minimum (nonredundant) sensor network design, in which there exists only one way of estimating each variable.
2. For a multicomponent process considered here, every stream contains all C components in nonzero concentrations. This implies that no component is completely separated out from any stream by a separator.
3. Either all or none of the compositions of a stream are measured, that is, partial stream composition measurement is not allowed.
4. A single sensor is used to measure all compositions of a stream.
5. Sufficient mass flows are measured such that it is possible to estimate all unmeasured mass flows using mass-flow measurements only.

Assumption 1 limits the scope of our work to the development of nonredundant sensor network design algorithms. However, these can serve as a starting point for the optimal placement of redundant sensors. Assumptions 2 to 4 are relevant only for multicomponent processes, and they enable us to quantify the minimum number of sensors required to estimate all variables. Although the basic design algorithms are developed under these assumptions, extensions to treat the general case are described later. The last assumption, which decomposes the bilinear problem into two coupled linear problems, disallows certain feasible sensor network designs in which mass flow variables are estimated using both mass flow and composition (or temperature) sensors. However, our experience shows that the sensor placement in most processes satisfies this assumption. In general, the estimation of a mass flow using both mass and composition sensors requires information from several sensors and leads to lower accuracy and reliability.

Reliability Evaluation of Bilinear Processes

We first consider the problem of sensor network design for a multicomponent process that consists of mixers and separators, but no splitters. The minimum number of sensors required to observe all variables and the evaluation of their reliabilities is discussed below.

Minimum number of sensors

When a multicomponent process consists of mixers and separators only, then a complete set of nonredundant equations that relate the variables are obtained by writing all the component balances for every unit and normalization equation for every stream. Generally, if only one of the compositions of a stream is unmeasured, then it can be indirectly estimated through the normalization equation. We have, however, assumed that compositions of a stream are not partially measured. Moreover, if the composition sensor of a stream fails, then all compositions of that stream become unmeasured and none of them can be indirectly estimated using the normalization equation. In effect, this nullifies the use of the normalization equation to indirectly estimate unmeasured compositions of a stream. Thus, only the overall flow and component balances around each unit are useful in indirectly estimating variables and, hence, are useful in computing reliabilities. A similar situation arises in multicomponent processes, where one of the components in all streams is unmeasurable. For example, in mineral beneficiation networks, the gangue component in all streams is unmeasured. In these networks, normalization equations are not useful in indirect estimation of mineral concentrations of streams.

Based on the preceding arguments, and making use of the last assumption, it can be proved that the minimum number of sensors required to observe all mass flows and compositions is equal to $2\times(e-n+1)$, where n is the number of process units (inclusive of the environment, which is considered as a unit) and e is the number of process streams. This follows from the fact that in a multicomponent process, there are $e\times(C+1)$ variables corresponding to mass flows and compositions, while there are $(n-1)\times(C+1)$ equations corresponding to flow and component balances around each unit except the environment. Thus, at least $(e-n+1)\times(C+1)$ variables have to be measured. Due to our assumption that all mass flows should be estimable using mass-flow measurements only, we require at least $e-n+1$ mass-flow sensors (Ali and Narasimhan, 1993). The remaining $(e-n+1)\times C$ variables can be measured using $e-n+1$ composition sensors, since each composition sensor can measure C components.

The preceding analysis also gives the feasible locations of sensors in order to uniquely observe all variables. For estimating mass flows using mass-flow measurements only, the streams with unmeasured mass flows should form a spanning tree (essential graph-theoretic terms are defined in the Appendix). Given that mass flows are measured or observable, we can use similar arguments to prove that in order to uniquely estimate every composition, streams with unmeasured compositions should form a spanning-tree structure.

Reliability evaluation

In the preceding section, it was shown that streams with unmeasured mass flows form a spanning tree, and streams with unmeasured compositions also form a spanning tree. Moreover, there is a unique way of estimating each variable.

Let T^m be the spanning tree formed by unmeasured mass-flow streams. Measured mass-flow variables are estimated directly through their measurements, and their reliabilities are equal, respectively, to the nonfailure probabilities of the sensors measuring them. Mass flows of streams that are unmeasured can be estimated through the fundamental cutsets of T^m. In this respect, the problem is identical to the one considered in Ali and Narasimhan (1993), and therefore the reliabilities of mass flows are given by,

$$R(M_j) = \prod_{\substack{i \in K_j^{fm} \\ i \neq j}} (1 - p_i^m) \tag{1}$$

where p_i^m is the failure probability of the mass-flow sensor of stream i, and K_j^{fm} is the fundamental cutset of spanning tree T^m corresponding to branch j.

In the case of compositions, we note that a single sensor is used to measure all compositions of a stream. Therefore, the reliabilities of measured compositions in a stream are all equal to the nonfailure probability of the composition sensor. Moreover, all unmeasured compositions of a stream are estimated indirectly using the same set of sensor measurements. Thus, all unmeasured compositions in a stream have the same reliabilities, which can be computed as follows.

A stream with unmeasured compositions will be a branch of T^x, the spanning tree formed by streams with unmeasured compositions. Let K_j^{fx} be the fundamental cutset with respect to this tree containing branch j. In order to estimate an unmeasured mass fraction, x_{jl}, of component l in stream j, the compositions of all streams in K_j^{fx} should be measured and mass flows of all streams in K_j^{fx} should be observable. The cutset K_j^{fx} must contain one or more streams with unmeasured mass flows. This follows from the fact that if mass flows of all streams in K_j^{fx} are measured, then it implies that all streams of cutset K_j^{fx} are chords of the spanning tree. This is impossible since a cutset cannot solely consist of chords of some spanning tree. Let $b_1, b_2, \ldots, b_k$ be the streams in K_j^{fx} that have unmeasured mass flows. It should be noted that $b_1, b_2, \ldots, b_k$ are the branches of spanning tree T^m. We identify the fundamental cutsets of T^m through which unmeasured mass flows of these streams can be estimated. If the cutsets are $K_1^{fm}, K_2^{fm}, \ldots, K_k^{fm}$, respectively, then the set, S_j, of mass-flow sensors required to estimate mass flows of all streams of K_j^{fx} is given by:

$$S_j = K_j^{fx} \cup K_1^{fm} \cup K_2^{fm} \cup \cdots \cup K_k^{fm} - \{b_1, b_2, \ldots, b_k\}. \tag{2}$$

Thus, to estimate composition x_{jl}, composition sensors of all other streams in K_j^{fx} and mass-flow sensors of all streams in S_j should be active. The reliability of x_{jl} is given by:

$$R(x_{jl}) = \prod_{\substack{i \in K_j^{fx} \\ i \neq j}} (1 - p_i^x) \times \prod_{i \in S_j} (1 - p_i^m) \tag{3}$$

where p_i^x is the failure probability of composition sensor of stream i.

Sensor Network Design Algorithms

The sensor network for multicomponent processes is designed so as to maximize the minimum reliability over all mass and composition variables. The problem, in our case, is to obtain that combination of spanning trees T^m and T^x that leads to maximum network reliability. First, we prove that the least reliability is always attained by an unmeasured composition as shown below.

Theorem 1. The minimum reliability is attained by an unmeasured composition in some stream and not by an unmeasured mass flow.

Proof. Let the least reliability among mass-flow variables be obtained for an unmeasured mass flow in stream k. With respect to spanning tree T^x, stream k may either be a chord or a branch, depending on whether the compositions of stream k are measured or not. In either case, stream k must be present in some fundamental cutset of T^x, since every branch and every chord of T^x must be present in some fundamental cutset. Let stream k be present in a fundamental cutset K_j^{fx}. Then, it follows from Eqs. 2 and 3 that in order to estimate compositions of stream j, all mass-flow sensors necessary for estimating the mass flow of stream k must be active, together with other mass-flow and composition sensors. Therefore, the reliability of x_{jl} is less than the reliability of mass flow in stream k. Either x_{jl} has the least reliability or some other unmeasured composition has lower reliability. In either case, the least reliability is attained by an unmeasured composition. From this result, it follows that while evaluating the network reliability, we need to evaluate the reliabilities of unmeasured compositions only.

We develop the solution strategy for three different cases, depending on the failure probabilities of mass-flow sensors and composition sensors and show how the spanning trees T^m and T^x are related for these cases.

Equal failure probabilities of mass-flow sensors and equal failure probabilities of composition sensors

The sensor network design for bilinear processes is relatively easy when all mass-flow sensors have the same failure probability, p^m, and all composition sensors also have the same failure probability, p^x. In this case, we prove that the optimal strategy is to locate the mass-flow sensor and composition sensor on the same set of streams. This implies that the spanning trees T^x and T^m are identical. Furthermore, we also show that the best spanning-tree solution for this case is identical to the optimal spanning tree obtained by considering only the mass flows of this process.

Theorem 2. In a multicomponent network, if the failure probabilities of mass-flow sensors are equal and those of composition sensors are also equal, then:

$$T^m = T^x = T^{m*}, \quad (4)$$

where T^{m*} is an optimal spanning tree of the pure mass-flow (linear) network of this process.

Proof. Let the pair of spanning trees $\{T^m, T^x\}$ represent a sensor network design solution for the bilinear process. We develop the proof by making use of the following lemma.

Lemma 1. If all mass-flow sensors have the same failure probabilities, then for any *given* T^x, the solution $\{T^m, T^x\}$, which maximizes network reliability is when T^m equals T^x.

Proof. Consider a fundamental cutset K_j^{fx} of the process graph with respect to a given T^x. The reliability of any composition x_{jl}, in stream j, is computed using Eqs. 2 and 3. From these equations, we can observe that for a given T^x, the only way of increasing the reliability of x_{jl} is to choose the spanning tree T^m such that the product $\Pi_{i \in S_j}(1-p_i^m)$ in Eq. 3 increases. For equal mass-flow sensor-failure probabilities, this is achieved by minimizing the cardinality of set S_j. Since every indirect estimation of a mass flow (using mass-flow measurements only) requires at least one other mass-flow sensor, the set S_j has least cardinality when the mass flows of all streams in K_j^{fx} (except stream j) are measured. In other words, K_j^{fx} should also be a fundamental cutset of T^m. This is valid for all fundamental cutsets of T^x, thereby implying that T^m and T^x should be identical.

The preceding lemma shows that we have to consider only solutions $\{T^m, T^x\}$, such that T^m and T^x are identical, in order to obtain one that maximizes network reliability. Since the failure probabilities of composition sensors are also equal, the reliability of a composition variable for any such solution computed using Eq. 3 reduces to:

$$R(x_{jl}) = (1-p^x)^c \times (1-p^m)^c, \quad (5)$$

where

$$c = |K_j^{fx} - 1|.$$

Thus, in order to maximize network reliability, we need to obtain a spanning tree such that the cardinality of its maximum cardinality fundamental cutset is less than or equal to the maximum cardinality fundamental cutset of any other spanning tree. If T^{m*} is the optimal spanning tree solution that maximizes the minimum reliability among mass-flow variables only, then this spanning tree clearly satisfies the preceding condition. Therefore, the optimal sensor network design for a bilinear process is $\{T^{m*}, T^x\}$ with T^x equal to T^{m*}. The solution T^{m*} can be obtained by applying algorithm SENNET (Ali and Narasimhan, 1993).

Equal failure probabilities of mass-flow sensors and unequal failure probabilities of composition sensors

It was proved in Lemma 1, that the spanning trees T^m and T^x in the optimal solution should be identical, if failure probabilities of all mass-flow sensors are equal. However, T^m need not be equal to the optimal mass-flow spanning tree, T^{m*}.

We illustrate this case through a three-component system, whose process graph is shown in Figure 1a. For this process we assume that the failure probabilities of all mass-flow sensors are equal to 0.1. The failure probabilities of composition sensors are indicated within parentheses for each stream in Figure 1a. The optimal mass-flow spanning tree, T^{m*}, is shown in Figure 1b. If all composition sensors have the same failure probability, then the best solution is to locate mass-flow and composition sensors on streams 3, 5 and 6. However, if we adopt the same solution for the present problem, the minimum reliability is 0.486, obtained for the compositions of streams 4. The optimal spanning-tree solution for this problem is shown in Figure 1c. By locating mass-flow and composition sensors on streams 6, 7 and 8, we get a higher network reliability of 0.605, corresponding to the reliability of compositions in streams 1, 2 and 3. This shows that the solution $T^m = T^x = T^{m*}$ is not optimal when composition sensor failure probabilities are unequal.

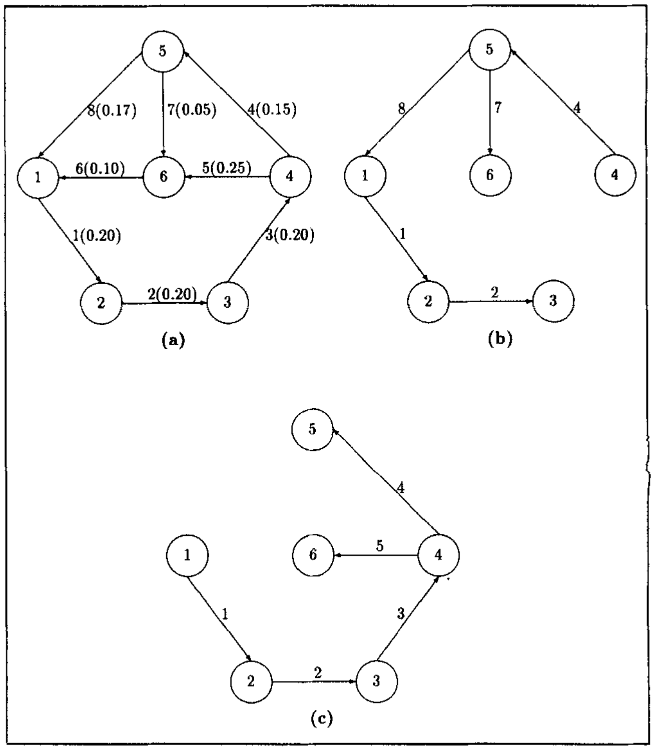

Figure 1. Optimal sensor networks of simplified ammonia process.

To obtain an optimal sensor network for this case, we modify SENNET slightly, as follows. We start with an initial spanning tree and use elementary tree transformation (Deo, 1974) at each iteration to obtain another spanning tree with better network reliability. The entering and leaving variable candidate sets are constructed as given in Ali and Narasimhan (1993), so that a spanning-tree structure is always maintained. The entering variable and the corresponding leaving variable are chosen such that the network reliability for the new solution is higher. The final spanning tree obtained from this algorithm corresponds to the mass flow and composition spanning trees.

Example 1

We use the simplified ammonia network shown in Figure 1a to illustrate the preceding algorithm. Mass-flow sensors are assumed to have a failure probability equal to 0.1, and failure probabilities of composition sensors are as shown in the figure. We start with the spanning tree $T^m = T^x = \{1, 2, 4, 7, 8\}$, shown in Figure 1b. The fundamental cutsets with respect to this tree are:

1. {**1**, 3}
2. {**2**, 3}
3. {**4**, 3, 5}
4. {**7**, 5, 6}
5. {**8**, 3, 6}

The minimum network reliability is 0.486, achieved by mass fractions in stream 4. The leaving variable candidates are those branches whose fundamental cutsets have at least one chord in common with the fundamental cutset of stream 4. Moreover, stream 4 may itself be a leaving variable candidate. Thus, the leaving variable candidate set is $O = \{1, 2, 4, 7, 8\}$. We arbitrarily select 7 as the candidate for outgoing variable. The corresponding set of entering variables I is {5}. We choose 5 as the entering variable since I has only one element. The new tree becomes {1, 2, 4, 5, 8}, giving a network reliability of 0.498. We observe that the network reliability improves; therefore, the choice of leaving and entering variables holds. Similarly, in the next iteration the leaving and entering variable candidates are 8 and 3, respectively, giving a new tree $T^x = T^m = \{1, 2, 3, 4, 5\}$ with a network reliability of 0.605. Since the network reliability does not improve any further for any choice of entering and leaving variables, we accept this as the final solution. Explicit enumeration of all possible spanning-tree solutions shows that this is also the globally optimal solution.

Unequal failure probabilities of mass-flow sensors and composition sensors with equal or unequal failure probabilities

When mass-flow sensors have unequal failure probabilities, the best sensor network design does not necessarily lead to identical spanning trees for T^m and T^x. This is shown by means of a trivial example.

Consider a simple process unit with one input stream and two output streams shown in Figure 2. The numbers within parentheses for each stream correspond to failure probabilities of mass-flow and composition sensors. Obviously, the optimal solution for this process is to measure mass flows of streams 1, 2, and compositions of streams 2 and 3, giving $T^x = \{1\}$ and $T^m = \{3\}$, which are not identical. Such a possibility arises whenever the reliability of a mass-flow variable is better if it is indirectly estimated than if it is measured. For example, if we measure mass flows of streams 1 and 2, then the reliability of estimating the mass flow of stream 3 is 0.56, which is higher than directly measuring it with a reliability of 0.50.

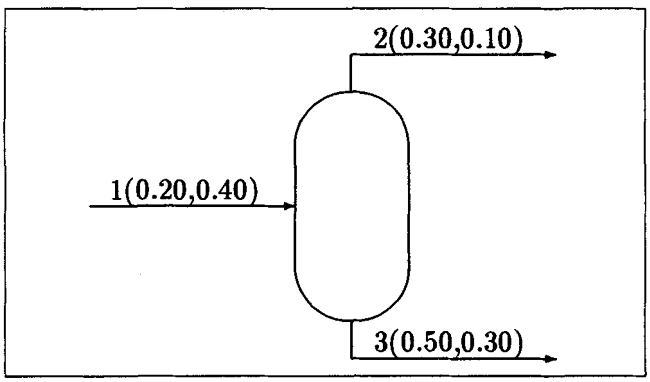

Figure 2. Unequal sensor failure probabilities example.

We now present an algorithm that can be used to obtain an optimal sensor-network design for this case.

Algorithm 3

We start with an initial spanning tree for T^x, and keeping this fixed, we attempt to obtain a spanning tree T^m that gives maximum network reliability. The spanning tree T^m is obtained as follows.

Phase 1

Step 1. Set $T^m = T^x$.

Step 2. Find unmeasured composition, x_{jl}, which has the least reliability.

Step 3. Obtain the set of streams, S_j whose mass-flow measurements are required to estimate x_{jl}.

Step 4. For each stream c_p of set S_j, identify all the fundamental cutsets of T^m that contain c_p as a member. Let $\{K_1^{fm}, K_2^{fm}, \ldots, K_r^{fm}\}$ be the set of such fundamental cutsets, corresponding to the branches $b_1, b_2, \ldots, b_r$ of T^m.

Step 5. If for any of the fundamental cutsets, K_q^{fm}, obtained in step 4,

$$1 - p_{c_p}^m \leq \prod_{\substack{i \in K_q^{fm} \\ i \neq c_p}} (1 - p_i^m),$$

then c_p should enter tree T^m and b_q should leave T^m.

Step 6. If no such cutset exists, stop; else go to step 2.

In the preceding algorithm, step 3 identifies all the mass-flow sensors (chords of T^m) that are required to estimate the least reliable composition variable. In step 4, we attempt to improve the network reliability by interchanging one of these chords with a branch of a fundamental cutset of which it is a member (elementary tree transformation of T^m). The network reliability improves if the indirect estimation of the mass flow of the chord is better than its direct measurement, which is checked in step 5. Phase 1 of the algorithm thus gives the spanning tree T^m corresponding to the given T^x that results in the maximum network reliability.

Phase 2. In this phase, we perform an elementary tree transformation of T^x and attempt to find a better solution $\{\bar{T}^x, \bar{T}^m\}$, where $\bar{T}^m$ is optimal for the chosen $\bar{T}^x$ as follows. The entering and leaving variables for T^x are obtained in the same manner as described in the previous section. For each choice of entering and leaving variable candidates, we get a spanning tree $\bar{T}^x$, corresponding to which the best possible mass-flow spanning tree can be obtained using phase 1. If this solution of $\{\bar{T}^x, \bar{T}^m\}$ gives an improved network reliability, we accept it and proceed with the next iteration of the algorithm; otherwise we try the next choice of entering and leaving variables for T^x. If none of these choices leads to an improvement in network reliability, we terminate the iterative process and accept the final solution.

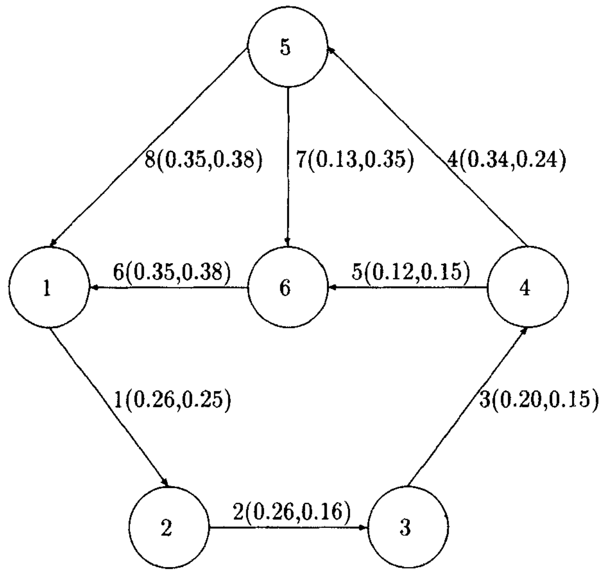

Figure 3. Ammonia process with unequal mass flow sensor failure probabilities.

Example 2

We consider the ammonia process shown in Figure 3. The failure probabilities of mass and composition sensors for each stream are respectively shown within parentheses in the figure. We choose an initial spanning tree $T^x = \{1, 2, 4, 7, 8\}$, as shown in Figure 1b. The optimal T^m corresponding to this T^x is obtained using the algorithm as follows:

Steps 1–2. Initialize $T^m = T^x = \{1, 2, 4, 7, 8\}$. The network reliability for this choice is 0.274, obtained for compositions in stream 8.

Step 3. The set S_8 is equal to $\{3,6\}$.

Step 4. Fundamental cutsets of T^m containing chord 3 are:

1. $K_1^{fm} = \{\mathbf{1}, 3\}$
2. $K_2^{fm} = \{\mathbf{2}, 3\}$
3. $K_4^{fm} = \{\mathbf{4}, 3, 5\}$
4. $K_8^{fm} = \{\mathbf{8}, 3, 6\}$

And the fundamental cutsets containing chord 6 are:

5. $K_7^{fm} = \{\mathbf{7}, 5, 6\}$
6. $K_8^{fm} = \{\mathbf{8}, 3, 6\}$

Step 5. We find that the fundamental cutsets to which 3 belongs do not give an indirect way of estimating mass flow of chord 3 with better reliability than its direct measurement. However, cutset K_7^{fm} provides an indirect way of estimating mass flow of 6 that is better than its direct measurement. So we interchange branch 7 with chord 6 and get a new $T^m = \{1, 2, 4, 6, 8\}$ giving an improved network reliability of 0.323. Further iteration does not lead to any improvement in the network reliability. Thus, the best T^m obtained for the initial T^x is $\{1, 2, 4, 6, 8\}$. The network reliability for this $\{T^x, T^m\}$ is 0.323 which is obtained for compositions in stream 8.

We now attempt to find an improved solution by elementary tree transformation of T^x. The leaving variable candidates can either be stream 8 or branches of T^x whose fundamental cutset has a common chord with K_8^{fx}. Thus, the leaving variable candidate set is {8, 1, 2, 4, 7}. We arbitrarily select edge 1 as the leaving variable and obtain the corresponding set of entering variables:

$$I = \{3,6,8\} - \{\{3,6,8\} \oplus \{1,3\}\} = \{3\}.$$

Since edge 3 is the only element of I, we choose it as an entering variable. The new T^x is {2, 3, 4, 7, 8}. Corresponding to this T^x, we find the optimal T^m, which happens to be identical. The network reliability for this trial solution is equal to 0.303. We observe that the network reliability does not improve, therefore, the choices of entering and leaving variable candidates are rejected. We continue till all candidates from set O have been verified, and find that the network reliability does not improve. Thus, we accept $T^x = \{1, 2, 4, 7, 8\}$ and $T^m = \{1, 2, 4, 6, 8\}$ as the solution. It should be noted that in this solution T^m and T^x are not identical.

Treatment of Splitter Units

Splitter units require special considerations in bilinear processes, since component balances around a splitter do not sufficiently describe this process unit. Instead, compositions of all streams incident on this unit have to be equated. Thus, if the compositions of some stream incident on a splitter are observable, then the compositions of all other streams incident on the splitter are also observable. Obviously, if splitter nodes are present in a process network, then the minimum number of composition sensors that are required to observe compositions in all streams is less than $e - n + 1$. Instead the minimum number of composition sensors is given by:

$$n_c = (e - n + 1) - \sum_i^{n_{sp}} e_{s_i} + 2 \times n_{sp}, \tag{6}$$

where e_{s_i} is the number of edges that are incident on splitter i and n_{sp} is the number of splitter nodes in a process. This equation can be obtained easily by observing that for splitter i, $e_{s_i} - 1$ composition equality constraints are imposed instead of a component flow balance. This gives $e_{s_i} - 2$ additional equations relating the compositions, leading to a corresponding reduction in the number of composition sensors. It should be noted that the preceding equation is applicable to process networks that do not contain splitters in series. Such a configuration may not occur in practical processes, since a single splitter can perform the same function as splitters in series.

Accounting for splitters in sensor network design

Clearly, when splitters are present, the streams with unmeasured compositions do not form a spanning tree, since less than $e - n + 1$ composition sensors are required to observe all mass fractions. However, we make some observations that enable us to exploit the structure of a spanning tree, and, hence, use SENNET to obtain a sensor network design. We assume that only one splitter is present in the process, and the extension to multiple splitters follows naturally from the arguments presented below.

To maintain the minimum number of composition sensors when splitters are present, some of the chords of T^x should also be unmeasured. The following three cases are possible, because at least one stream incident on any node should be part of a spanning tree.

Case 1. Only one of the streams incident on the splitter is a branch of T^x.

Case 2. Exactly two of the streams incident on the splitter are branches of T^x.

Case 3. More than two streams incident on the splitter are branches of T^x.

The spanning tree T^x and associated chords for the three cases are shown in Figure 4a, 4b and 4c, respectively. In these figures, the node marked S represents the splitter unit. The solid lines are the branches of T^x and the dotted lines are the chords. Only the splitter branches and chords that are part of the fundamental cutsets of T^x (corresponding to splitter branches) are shown. The streams incident on the splitter are denoted as s_i. The parts of T^x that are connected to the splitter branches are depicted as subtrees without explicitly showing the branches or chords, because these are not pertinent to the following discussion.

For case 1, the fundamental cutset corresponding to the splitter branch, is identical to the cutset that separates the splitter unit from the rest of the process. Thus, this fundamental cutset contains only the splitter streams as chords, as shown in Figure 4a. Clearly, to estimate the compositions of all the splitter streams, a composition sensor must be placed on only one of the splitter streams in this fundamental cutset, while the composition of the remaining streams should be unmeasured.

For case 2, both the fundamental cutsets, $K_{s_1}^{fx}$ and $K_{s_2}^{fx}$, contain the same nonsplitter streams as chords, $c_1, c_2, \ldots, c_r$ as shown in Figure 4b. In this case, we can assume, without loss of generality, that compositions of $c_1, c_2, \ldots, c_r$ are measured and no splitter stream compositions are measured. This follows from two observations. First, if compositions of $c_1, c_2, \ldots, c_r$ and one splitter stream are measured, then the compositions of all streams in $K_{s_1}^{fx}$ and $K_{s_2}^{fx}$ become redundant, which violates the minimum number of sensors requirement. Second, if compositions of a splitter stream are measured and compositions of one of the chords, say c_p, are unmeasured, then we can perform an elementary tree transformation by interchanging c_p with branch s_1 or s_2. This will have no effect on the reliabilities of variables, because this interchange involves two unmeasured streams and no sensor relocation. The result of this interchange gives a new spanning tree corresponding to case 1.

We show that all T^x trees corresponding to case 3 can be transformed to trees corresponding to case 2, without any change in reliabilities. As an example, Figure 4c shows a T^x with three splitter streams as branches and their fundamental cutsets. Using arguments similar to case 2, we can assume that none of the splitter streams have measured compositions. Furthermore, one of the nonsplitter chords, say c_p, belonging to $K_{s_1}^{fx}$, $K_{s_2}^{fx}$, or $K_{s_3}^{fx}$ should have unmeasured compositions in order to ensure that compositions are not redundant. As before, we can perform an elementary tree transformation by interchanging c_p with a splitter branch, without any change in reliabilities.

The net result of the preceding analysis is that we can consider spanning trees for T^x containing one or two splitter branches. If T^x contains only one splitter branch, one of the splitter stream compositions must be measured. In this case, among all the splitter streams the one whose composition

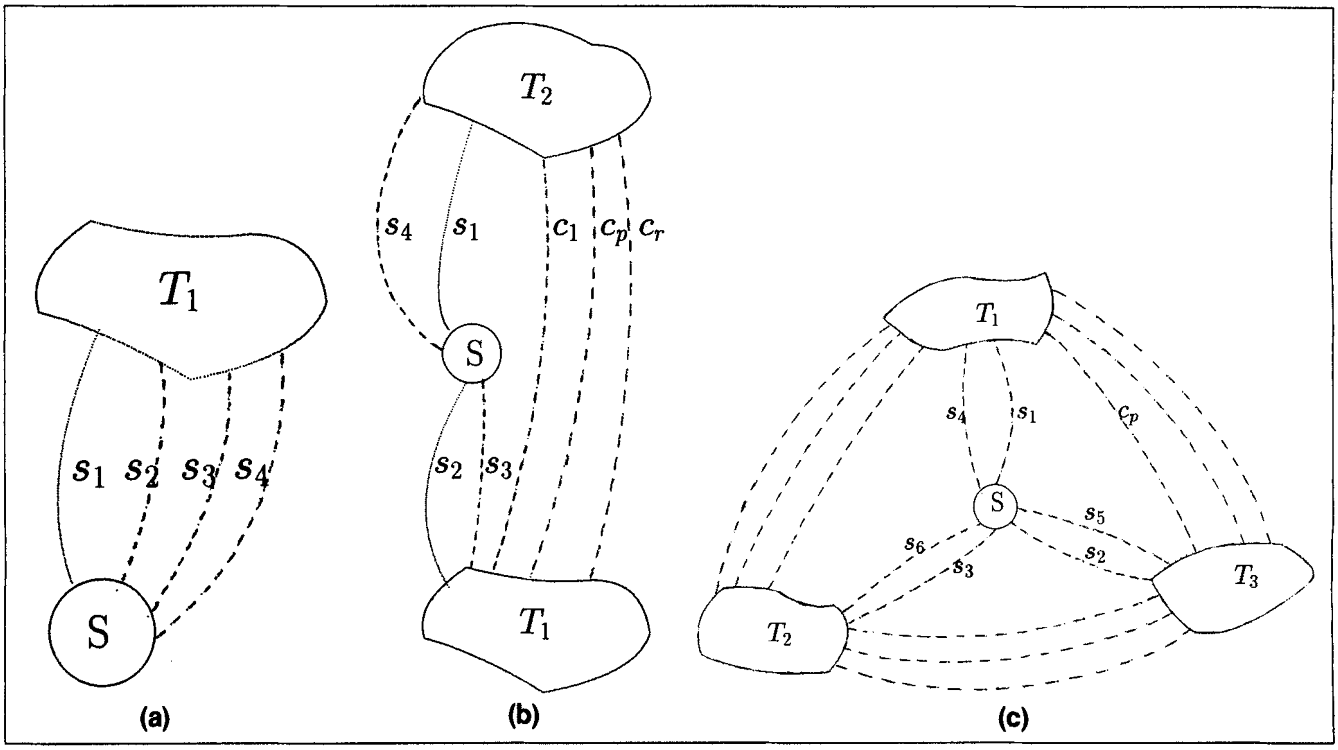

Figure 4. Sensor network design with: (a) one splitter edge as branch; (b) two splitter edges as branches; (c) more than two splitter edges as branches.

sensor has the least failure probability is measured. If T^x contains two splitter streams as branches, then none of the splitter streams have measured compositions. It is now possible to use the sensor network design algorithm for the three different cases described earlier with the following modifications.

1. When splitters are present, it can be proved using Lemma 1 that, in the optimal sensor network design, T^x and T^m are identical for equal failure probabilities of mass sensors, only if T^x contains one splitter stream. Since, in general, it cannot be determined whether T^x and T^m should be identical in the optimal solutions, algorithm 3 is always used for any given failure probabilities of mass flow and composition sensors. Moreover, the initial spanning tree and entering variable set for T^x is chosen such that among streams incident on any splitter, not more than two are branches of T^x. This can be easily incorporated in the spanning tree generation algorithm. At each iteration, we also choose the entering variable such that not more than two streams incident on the same splitter become branches of T^x.

2. In order to compute the reliability of a composition variable, we need to identify all mass-flow sensors and all composition sensors that are useful for its estimation. These are obtained as follows.

• If T^x contains only one stream of any splitter, then this implies that the compositions of one of the splitter streams that has the least failure probability, say p_s^x, is measured. Thus, the composition reliabilities of all streams incident on that splitter are equal to $(1-p_s^x)$. If the fundamental cutset of any other stream, K_j^{fx} contains one or more streams of a splitter then in reliability computation of composition x_{jl}, the composition of all these splitter streams is assumed to be measured using one composition sensor with failure probability p_s^x.

• If T^x contains two streams of a splitter, say s_1 and s_2, then none of the splitter stream compositions are measured. Instead, compositions of a splitter stream are indirectly estimated using the fundamental cutset, $K_{s_1}^{fx}$ or $K_{s_2}^{fx}$. Let $K_{s_1}^{fx}$ be the cutset through which compositions of a splitter stream are estimated. Let $c_1, c_2, \ldots, c_r$ be the nonsplitter chords of $K_{s_1}^{fx}$. The set of mass-flow measurements required to estimate splitter stream compositions is obtained as before using Eq. 2. However, only the composition sensors of chords $c_1, c_2, \ldots, c_r$ are required for estimating splitter stream compositions. Thus, the reliabilities of all splitter stream compositions are given by:

$$R(x_{s_i l}) = \prod_{\substack{k \in K_{s_1}^{fx} \\ k \neq s_i}} (1-p_k^x) \times \prod_{k \in S_{s_1}} (1-p_k^m). \tag{7}$$

If the fundamental cutset of any other stream, K_j^{fx} contains one or more streams of a splitter, then the reliability of x_{jl} also depends on the mass-flow and composition sensors required to estimate compositions of splitter streams. If the set S_j is constructed according to Eq. 2, then the reliability of x_{jl} is given by the following equation:

$$S_j^* = S_j \cup S_{s_1} \tag{8}$$

$$Z_j = K_j^{fx} \cup \{c_1, c_2, \ldots, c_r\} - \{j\} \tag{9}$$

$$R(x_{jl}) = \prod_{i \in Z_j} (1-p_i^x) \times \prod_{i \in S_j^*} (1-p_i^m), \tag{10}$$

where S_j^* is the set of mass-flow sensors and Z_j is the set of composition sensors required to estimate composition in stream j.

It was proved in Theorem 1 that the least reliability is attained by an unmeasured composition. However, if splitters are present, there are rare instances when an unmeasured mass flow of some splitter stream may have lower reliability than all unmeasured compositions. This may occur, for instance, if a splitter stream j is a branch both in T^m and T^x, and if one of the splitter streams has measured composition. In this case, the unmeasured mass flow of stream j will not be useful for estimating any of the unmeasured compositions and Theorem 1 cannot be used. In order for the mass-flow of stream j to have the least reliability, the mass-flow failure probabilities of splitter streams must be substantially higher than other mass-flow and composition sensor failure probabilities. We therefore, have ignored this case in our development.

Example 3

In this example, we illustrate the design of a sensor network when splitters are present in the process. Consider the ammonia network given in Figure 1. For reasons of simplicity we assume that all mass-flow sensors have equal failure probability of 0.10 and all composition sensors also have equal failure probability of 0.20. We also assume that only a single splitter is present in the process that is represented by node 5.

To observe mass flows and compositions of all streams, the minimum number of mass-flow sensors is three, whereas that of composition sensors is $8-6+1-3+2=2$ (Eq. 6). We start with an initial tree $T^x = T^m = \{2, 3, 4, 5, 8\}$. Note that we cannot start with the spanning tree given in Figure 1b, as it contains three splitter branches. The fundamental cutsets of this spanning tree are:

1. {**2**, 1}
2. {**3**, 1}
3. {**5**, 6, 7}
4. {**4**, 1, 6, 7}
5. {**8**, 1, 6}

Since T^x contains two splitter branches, all splitter stream compositions are unmeasured. Thus, the compositions of stream 7 (a splitter edge) is unmeasured. However, the mass flow of this stream is measured. Reliabilities of compositions for all streams, except for 4, 5 and 7, can be computed using Eqs. 2 and 3, since the fundamental cutsets of these streams in T^x do not contain any splitter stream.

The compositions of splitter stream 8 are observable using the mass and composition measurements of streams 1 and 6 (fundamental cutset 5) with a reliability of 0.518. Therefore, the compositions of splitter streams 4 and 7 are also observable with the same reliability. Compositions of stream 5 can be estimated using the mass and composition measurements of streams 6 and 7 (fundamental cutset 3). However, the compositions of stream 7 are indirectly estimated using mass-flow and composition sensors of streams 1 and 6. Therefore, the compositions of stream 5 are observable through mass-flow measurements of streams 1, 6, 7 and the composition measurements of streams 1 and 6, with a reliability of 0.467.

Corresponding to this T^x we obtain the optimal T^m using Algorithm 2. We observe that T^m and T^x are identical. The network reliability is 0.467, which is attained by compositions of stream 5. As in SENNET (Ali and Narasimhan, 1993), we find the set of outgoing variables is $O = \{2, 3, 4, 8\}$. If edge 4 is chosen as the leaving variable candidate, the corresponding set of entering variables is $I = \{6, 7\}$. If we choose either 6 or 7 as the entering variable, it is found that the network reliability does not improve. However, the network reliability improves if edges 8 and 1 are chosen as the candidates for the leaving and entering variables. This gives a new spanning-tree solution corresponding to the edges {1, 2, 3, 4, 5}. Further iterations do not improve the network reliability. Explicit enumeration of all solutions shows that this solution is also the global optimum.

Energy Networks

The algorithms in the previous sections were developed for multicomponent processes. In this section, we show that the same algorithms can also be applied to heat-exchanger networks, which consist of heat exchangers, splitters and mixers.

In order to represent processes containing heat exchangers as process graphs, Stanley and Mah (1977) and Kretsovalis and Mah (1988) represent a heat exchanger by two nodes (equivalent to the tube and shell sides of the exchanger), connected by an *artificial pure energy stream* that indicates the energy transfer. This representation is not conducive for sensor network design due to the need for keeping track of the real and artificial streams. Instead, we use two different graphs to represent energy networks. A *mass flow graph*, G^m, is used to represent the flow balances of the process and is derived from the flowsheet by replacing each exchanger by two disjoint nodes representing the tube and shell sides of the exchanger, and adding an environment node to which all inputs and outputs of the process are connected. The energy balances are captured by an *energy flow graph*, G^e, which is directly derived from the flowsheet by adding an environment node to which process feeds and products are connected. As an example, the flowsheet of a crude preheat train of a refinery is shown in Figure 5a. The energy flow graph for this process is identical to Figure 5a, except for the environment node, which is not shown for clarity. The mass-flow graph is shown in Figures 5b and 5c. Figure 5b shows the mass-flow graph of cold fluid (crude oil), and Figure 5c shows the mass-flow graph of the hot fluids. It should be noted that Figures 5b and 5c are connected only through the common environment node, while the nodes representing the heat exchangers are not directly linked.

All the algorithms developed for multicomponent networks can now be applied to obtain the optimal placement of flow and temperature sensors. The streams with unmeasured flows will form a spanning tree of G^m, while the streams with unmeasured temperatures will form a spanning tree of G^e.

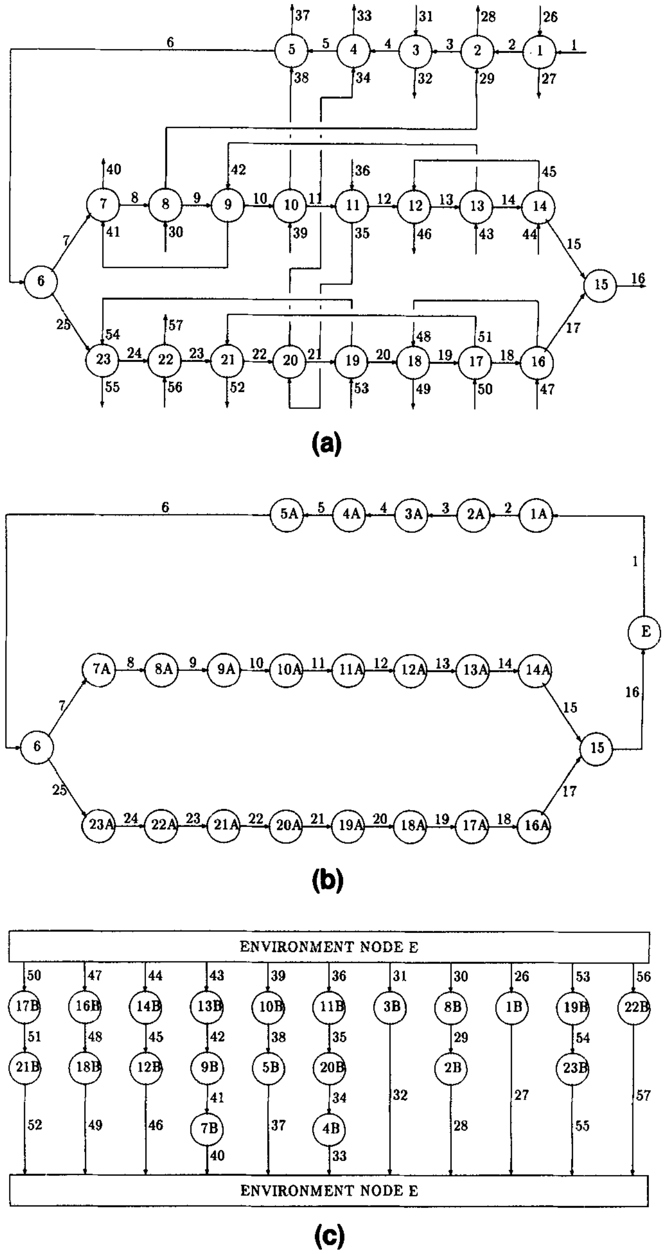

Figure 5. (a) Crude preheat train of a refinery; (b) mass flow of cold fluid; (c) mass flow of hot fluids.

Extensions to Sensor Network Design

In the preceding algorithms, we have separately considered multicomponent processes and energy networks. It is possible to simultaneously consider multicomponent processes with energy transfer (that is, the sensor network design problem involving flow, temperature, and composition sensors), provided we assume that specific enthalpy is a function of temperature only and is not dependent on composition. In order to solve this problem, we make use of Algorithm 3. We start with a spanning-tree solution for temperatures, T^e, and a spanning-tree solution for compositions, T^x. Corresponding to these, the best spanning tree for mass flows, T^m, which maximizes the minimum reliability among all compositions and temperatures, is obtained as described in phase 1. Keeping T^m fixed, we obtain the best T^x, and T^e using phase 2 of the algorithm. The procedure can be repeated until the network reliability does not improve any further.

The sensor network design algorithms for multicomponent processes have been developed under assumptions 2 to 4 listed in the problem-definition section. The sensor network design algorithms can be extended as follows to treat the general case when these assumptions do not hold. If a component is not present in a stream, then we can equivalently assume that the component is present in the stream, but is always measured in every sensor network design with a sensor having zero failure probability. Similarly, if a stream consists of only one component, then all components are assumed to be present in the stream, but measured with a sensor having zero failure probability. Thus, the "measured" compositions of a stream may have different failure probabilities for different components. Similarly, if more than one sensor is used to measure the compositions of a stream, then they may have different failure probabilities. In addition, if we allow partial stream measurement, then the sensors required to estimate the components of a stream can be different for different components, which implies that the compositions of a stream can have different reliabilities. If we ignore all sensor network designs in which normalization equations are useful in estimating unmeasured compositions, then, in the nonredundant sensor network design, the unmeasured compositions of each component j forms a spanning tree, T^{x_j}, which may be different for different components.

The preceding problem can again be solved using Algorithm 3. We start with a set of spanning trees corresponding to the solutions T^{x_j} and find the spanning tree T^m that maximizes the minimum reliability among all compositions, as in phase 1. Keeping T^m fixed, we apply phase 2 of the algorithm to modify each T^{x_j}, such that each T^{x_j} is optimal for the current T^m. The procedure is repeated until no further improvement in network reliability is obtained.

Results and Discussion

In this section, we present simulation results on realistic industrial processes to demonstrate the performance of the algorithm. We ignore the case of equal-failure probabilities of mass and equal-failure probabilities of composition sensors since these results will be similar to those presented in our previous article (Ali and Narasimhan, 1993). First, we discuss the results for equal-mass and unequal-composition failure probabilities. This is illustrated in the following two processes.

Example 4

Consider the mineral beneficiation circuit shown in Figure 6. This network contains 9 nodes and 17 edges. Since, this network does not contain any splitter node, the minimum number of mass and composition sensors is equal to nine each. All mass-flow sensors are assumed to have a failure probability of 0.10, whereas failure probabilities of composition sensors are given in Table 1. Since all mass-flow sensors have the same failure probability, the optimal sensor network design corresponds to $T^x = T^m$. As described earlier, SENNET was applied to obtain a sensor network design. The results of simulation are given in Table 1. The problem was also solved using explicit enumeration, and these results are also shown. The number of possible spanning trees is 7380. Since any of the spanning trees can be a choice for T^m or T^x, the total number of (T^x, T^m) combinations is more than 50 million. Out of these only 15 solutions (corresponding to $T^x = T^m$) are optimal, giving a network reliability of 0.243 (obtained for compositions in stream 14).

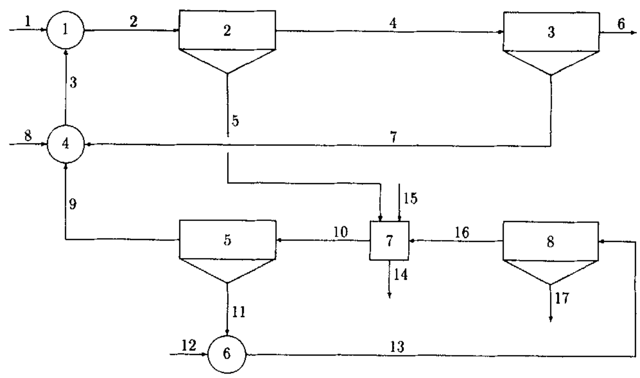

Figure 6. Mineral beneficiation circuit.

The initial tree is given in the first column of Table 1, and the optimal solution obtained by SENNET is given in column 2 for five different choices of the initial solutions. In all of the cases, the solutions obtained by extended SENNET were found to be globally optimal.

Example 5

Another example we have chosen is the organic synthetic juice extraction plant (Meyer et al., 1990) shown in Figure 7. This process contains a splitter node denoted by S in the figure. We find that a minimum of seven mass-flow and six composition sensors are required for this process. The results of applying extended SENNET to this process are shown in Table 2, along with the failure probabilities of the sensors. Although for this case the failure probabilities of all mass-flow

Table 1. Data and Results of Mineral Beneficiation Circuit (Case 2)

Data
Number of units: 9
Number of streams: 17
Number of splitters: 0
Failure probabilities of mass flow sensors: 0.10
Failure probabilities of composition sensors:
0.263 0.340 0.175 0.096 0.340 0.157
0.316 0.319 0.062 0.165 0.290 0.147
0.054 0.293 0.154 0.203 0.112

Results
Minimum number of mass flow sensors: 9
Minimum number of composition sensors: 9
Number of spanning trees: 7,380
Number of optimal solutions: 15
Optimal network reliability: 0.243

Initial T^x, T^m	Optimal T^x, T^m
1 2 3 4 5 9 11 15	1 2 6 8 11 12 13 14
2 3 7 9 11 13 15 16	1 2 6 7 11 13 14 17
1 3 4 5 7 10 11 13	1 2 6 8 11 13 14 17
1 2 6 8 9 11 15 16	1 2 7 8 11 12 14 17
1 4 7 8 11 12 14 17	1 2 7 8 11 12 14 17

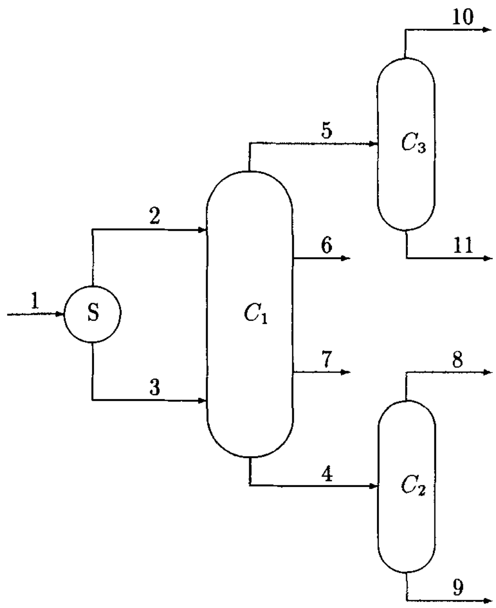

Figure 7. Synthetic juice extraction plant.

sensors are equal, we seek a solution, using the algorithm described for unequal mass-flow sensor failure probabilities, since the process contains a splitter unit. Furthermore, T^x is allowed to contain at most two splitter streams as branches.

Table 2 shows that the number of spanning trees (found by explicit enumeration) for this process is 667. Any of these spanning trees can be chosen for T^m. However, for T^x only 642 spanning trees that have one or two splitter edges as branches can be chosen. Thus, the total number of (T^x, T^m) combinations is 428,214. Out of these, only four solutions are optimal, with a network reliability of 0.272. The solutions obtained by extended SENNET for five different choices of initial solution are shown in Table 2. All of these were found to be globally optimal. Although it could not be theoretically determined whether in the optimal solutions T^x and T^m are identical when mass-flow sensors have equal failure probabilities and the process contains splitter units, the simulation results for this example depicts that T^x and T^m are identical. It was also observed that in all of the spanning-tree solutions obtained by the algorithm, only one splitter stream is a branch, implying that compositions of one of the splitter streams are measured. Since the composition sensor of splitter stream 3 has the least failure probability, it is measured. Generally, for maximum network reliability, compositions of one of the splitter streams are measured. This can be observed from Eqs. 8–10, which show that the number of mass-flow and composition sensors required to estimate the compositions of a stream j can be large, if its fundamental cutset contains a splitter stream whose compositions are indirectly estimated.

Table 2. Data and Results of Synthetic Juice Extraction Plant

Data
Number of units: 5
Number of streams: 11
Number of splitters: 1
Failure probabilities of mass flow sensors: 0.10
Failure probabilities of composition sensors:
0.263 0.340 0.175 0.096 0.340 0.157
0.316 0.319 0.062 0.165 0.290

Results
Minimum number of mass flow sensors: 7
Minimum number of composition sensors: 6
Number of spanning trees: 667
Number of optimal solutions: 4
Optimal network reliability: 0.272

Initial T^x	Optimal T^x	Optimal T^m
1 5 8 10	2 7 8 11	2 7 8 11
1 5 9 10	2 7 9 11	2 7 9 11
1 2 9 11	2 7 9 11	2 7 9 11
1 2 8 10	2 7 8 11	2 7 8 11
3 4 8 10	2 7 8 10	2 7 8 10

Example 6

A mineral beneficiation process (Figure 6) is used to demonstrate the performance of our design algorithm when mass and composition sensor failure probabilities are unequal. These results are presented in Table 3 for the failure probabilities. The spanning-tree solutions for T^x and T^m obtained by our algorithm are shown in columns 3 and 4, respectively, for five different initial choices of T^x. The network reliability for the initial solution, taking T^m to be identical to T^x, is shown in column 2, and the network reliability of the best solution obtained by our algorithm is shown in the last column. From the fact that our algorithm did not give solutions with identical network reliability for all choices of the initial solution, it can be inferred that the algorithm does not give globally optimal solutions in all cases. Since explicit enumeration of all solutions is not possible, we cannot quantify how suboptimal these solutions are. However, we find by explicit enumeration the best spanning-tree solution for T^x equal to T^m, which gives a network reliability of 0.127. Compared to this, the solutions obtained by our algorithm have higher network reliability. Therefore, we conclude that our algorithm is generating reasonably good designs.

Example 7

Finally, we demonstrate the performance of the algorithms, on the crude preheat train shown in Figure 5a. This network contains 24 nodes and 57 edges, with node 6 being the splitter node. The number of mass-flow sensors required to observe all mass flows is 13, whereas the number of temperature sensors required is 33. It has been assumed that the failure probabilities of mass-flow sensors are equal to 0.10. The failure probabilities of temperature sensors are assumed to be unequal and are given in Table 4.

It is practically impossible to evaluate all the solutions of this network and compare the results. However, we have simulated the network using different initial solutions. For all of these we obtain the sensor network designs with a network reliability of 0.227. The near optimality of the solutions can be justified by the following observations. In each initial solution, the least reliability variable was estimated using 20 temperature sensor measurements and five mass-flow sensor measurements; whereas the least reliability variable in the optimal solution was estimated using only five temperature and two mass-flow measurements. Moreover, in the network we cannot generate a spanning tree that will have a funda-

Table 3. Data and Results of Mineral Beneficiation Circuit (Case 3)

Data
Number of units: 9
Number of streams: 17
Number of splitters: 0
Failure probabilities of mass flow sensors:
0.164 0.107 0.099 0.153 0.330 0.088 0.139 0.266 0.231
0.108 0.228 0.332 0.173 0.357 0.290 0.206 0.119
Failure probabilities of composition sensors:
0.263 0.340 0.175 0.096 0.340 0.157 0.316 0.319 0.062
0.165 0.290 0.147 0.054 0.293 0.154 0.203 0.112

Results
Minimum number of mass flow sensors: 9
Minimum number of composition sensors: 9
Number of spanning trees: 7380

Initial T^x	Init. Rel.	Optimal T^x	Optimal T^m	Opt. Rel.
2 3 7 9 11 13 15 16	0.026	1 2 7 8 9 12 14 16	1 5 7 8 9 12 14 16	0.141
1 3 4 5 7 10 11 13	0.027	1 2 6 7 11 12 13 14	1 5 6 7 11 12 13 14	0.137
1 2 6 8 9 11 15 16	0.069	1 2 6 8 9 11 14 16	1 5 6 8 9 11 14 16	0.137
1 4 7 8 11 12 14 17	0.090	1 2 7 8 11 12 14 17	1 5 7 8 11 12 14 17	0.141
1 2 3 4 5 9 11 16	0.005	3 4 6 8 11 12 14 16	3 5 6 8 11 12 14 16	0.137

mental cutset less than cardinality four. This conclusion is drawn from the fact that each node in the network has four or more edges incident on it, except the splitter and mixer nodes. Therefore, in any solution we require at least one flow and three temperature measurements in order to estimate every unmeasured temperature. Since, in our design the least reliable temperature variable requires only two more temperature and one more flow measurement, we can conclude that our design is reasonably good. The algorithm requires only 8–10 seconds of computation time on a PC 386 with coprocessor.

From the preceding simulation results, we observe that in the optimal solutions the network reliability is very low in all cases. This is due to the fact that only the minimum number of sensors is used to design the sensor network. In order to observe the variables with high reliability, it is necessary to install redundant sensors. Thus, the problem of redundant sensor network design for bilinear processes is important and needs to be tackled.

Concluding Remarks

In this article, we have addressed the problem of sensor locations for bilinear processes, taking reliability issues into

Table 4. Data and Results of Crude Preheat Train

Data
Number of units: 24
Number of streams: 57
Number of splitters: 1
Failure probabilities of mass flow sensors: 0.10
Failure probabilities of temperature sensors:
0.263 0.340 0.340 0.157 0.165 0.054 0.293 0.120 0.107 0.099
0.086 0.231 0.332 0.173 0.290 0.206 0.119 0.172 0.061 0.099
0.381 0.268 0.230 0.230 0.164 0.357 0.276 0.175 0.096 0.164
0.316 0.281 0.319 0.062 0.266 0.139 0.290 0.147 0.330 0.203
0.112 0.153 0.256 0.157 0.108 0.228 0.250 0.270 0.209 0.203
0.152 0.085 0.338 0.365 0.164 0.276 0.291

Results
Minimum number of temperature sensors: 33
Minimum number of mass flow sensors: 13

Optimal locations for mass sensors
1, 25, 26, 30, 31, 33, 37, 40, 44, 49, 50, 53, 56

Optimal locations for temperature sensors

Initial T^e	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17 18 19 20 21 22 23 24
Optimal T^e	1 2 7 10 15 16 18 22 29 31 33 36 37 39 40 43 46 47 49 52 53 55 56 **25**
Initial T^e	1 2 3 4 5 6 7 8 10 11 12 14 15 17 18 19 20 21 22 23 24 41 42
Optimal T^e	1 2 7 8 13 15 18 19 22 31 33 36 37 39 40 41 43 44 47 52 53 55 56 **25**
Initial T^e	1 2 3 4 5 8 9 10 11 12 13 15 17 18 19 20 22 23 24 25 34 38 45
Optimal T^e	1 2 7 8 10 12 17 19 22 31 33 38 39 40 43 44 46 47 50 52 53 55 56 **25**
Initial T^e	1 2 3 4 5 6 7 10 11 12 13 14 15 17 18 19 20 22 23 24 25 29 34 51
Optimal T^e	1 2 7 10 15 16 18 22 29 31 33 36 37 39 40 43 46 47 49 52 53 55 56 **25**
Initial T^e	2 3 4 5 7 8 10 11 12 14 15 17 18 20 22 23 24 29 35 38 39 45 48
Optimal T^e	1 2 7 10 15 16 18 22 29 31 33 36 37 39 40 43 46 47 49 52 53 55 56 **25**

consideration. Although algorithms are developed for designing a nonredundant sensor network, they can serve as a starting point for the design of redundant sensor networks for generalized processes. Considerable scope exists for developing sensor network design algorithms to meet other desired objectives such as estimation accuracy, controllability, and safety. These can form part of an integrated PID development package.

Notation

c = cardinality of a fundamental cutset
c_i = chord i of a spanning tree
c_p = entering variable candidate
n_c = minimum number of composition sensors
$R(i)$ = reliability of variable i

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Appendix: Definitions of Important Graph-Theoretic Concepts

Graphs and subgraphs

A *graph* consists of a set of nodes, V, and a set of edges, E. Each edge is associated with a pair of nodes, which it joins. An example of a graph is shown in Figure 1a, which has six nodes drawn as circles and eight edges shown as lines. In general, a process graph can simply be obtained from the process flowsheet by adding an additional node called the environment node to which all process feeds and products are connected. If the directions of the edges are ignored, then an undirected graph is obtained; otherwise the graph is directed.

A *subgraph* of a graph consists of a subset of nodes and edges of the graph. Each edge of the subgraph joins the same two nodes as it does in the graph. In other words, if an edge is part of a subgraph, then the end nodes with which it is associated in the graph should also be part of the subgraph. Figures 1b and 1c are subgraphs of the graph in Figure 1a.

Paths, cycles, and spanning trees

A *path* between two nodes is a connected sequence of edges between the nodes, with no edge repeated twice. A closed path is called a *cycle*. For example, edges 1, 2 and 3 form a path between nodes 1 and 3, while edges 4, 5 and 7 form a cycle.

A graph is *connected* if there exists a path between every pair of nodes. A connected subgraph of the graph that does not contain any cycles and that includes all vertices of the graph, is called a *spanning tree* of the graph. An edge of the graph that is part of the spanning tree is called a *branch*, while edges of the graph not part of the spanning tree are called *chords*. Figures 1b and 1c are examples of spanning trees of the graph of Figure 1a. Corresponding to the spanning tree of Figure 1a, edges 1, 2, 4, 5 and 8 are branches, while the remaining edges 3, 6 and 7 are chords.

Cutsets and fundamental cutsets

A *cutset* of a graph is a minimum set of edges of the graph, whose removal disconnects the graph. A cutset of the graph that contains only one branch of a spanning tree and zero or more chords is called a *fundamental cutset* corresponding to the spanning tree. For example, edge sets {1,2}, and {1,3} are cutsets of the graph of Figure 1a. However, {1,3} is also a fundamental cutset corresponding to the spanning tree of Figure 1b.

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